Communication Theory II

Lecture 2: Review on Fourier analysis of signals and systems

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Quiz 1

- In a blank paper write your **name** and **section number** and **list one advantage of digital communication systems**
- Time: 2 min

Fast Overview

Required Course Materials (updated)

Text books	• Simon Haykin, "Digital Communication Systems, 1st Edition," Wiley, 2013
Software	MATLAB
Other materials	• B. P. Lathi and Zhi Ding, "**Modern Digital and Analog Communication Systems, 4th Edition**" Oxford Series in Electrical and Computer Engineering, 2009. • S. Haykin, M. Moher, "**Communication Systems, 5th Edition,**" Wiley, 2009. • Michael Fitz, "**Fundamentals of Communications Systems (Communications Engineering)**," McGraw-Hill Professional, 2007 • Robert G. Gallager, "**Principles of Digital Communication**," Cambridge University Press, 2008. • MIT Course Number 6.450: Principles of Digital Communications I http://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-450-principles-of-digital-communications-i-fall-2006/index.htm

Grading

Assessment Method	Weights	Time
Mid Term Examination	20	Week 7
Oral Examination	10	Week 12
Semester work	20	Every week
Final Term Examination	100	Week 14
Total	150	

Oral Exam (10 marks)

- Objectives
  - Ability to present your self and your ideas
  - Ability to enhance creativity of individuals
  - Ability to enhance searching capabilities of the students
  - Ability to work in team

Oral Exam (Cont'd)

- Each section will be divided into groups, each group is 3 to 5 persons
  - Group selection (within the section) is optional till March 1st. After, the group will be assigned by the alphabetic order

Oral Exam (Cont’d)

- Each group will design a suggested solution for a problem in digital communications and present the solution in 4 slices as follow:
  - First slice contains the title and the student names
  - Second slice contains the problem to be solved and the traditional solution
  - Third slice contains a sketch of the proposed solution and/or needed materials
  - Last slice contains the reference list used in the project
  - An additional slice can be added containing a simulation if needed

Oral Exam (Cont’d)

- Each group will take a 8-10 min presentation exam, each student have to describe his role in the project
- Two minutes for questions
- Oral exam is optional for students with excellent contributions during the lectures or sections

Hints for a successful presentation

- Work as a group: distribute tasks
- Read well about the topic
- A good presentation is like a story. The presenter goal is to present a connected lines for the story, while grapping the attention of the audience
- Prepare yourself well, most importantly, **train well**

Semester work (20 marks)

- **Goal:** Test the work of the students during the semester
  - **Online quiz/homework** [open after the lecture (Thursday, 12 PM, noon) and closed before the section (Wednesday, 8 AM)]
  - **Small Quizzes** inside the lectures and/or sections

Course Website

- http://lms.mans.edu.eg/eng/
- The site contains the lectures, quizzes, homework, and open forums for feedback and questions
- Log in using your name and password
- Password for quizzes: third

Course Contents* (updated)

- Introduction to digital communications
- Mathematics of digital communications
  - Fourier analysis of signals and systems
  - Probability theory and Bayesian Interface
  - Stochastic Processes
  - Information theory
- From analog to digital communications
  - Waveform Coding Systems
  - Digital Modulation/Demodulation
- Signaling over AWGN, band-limited, and fading channels
- Error control coding

* 10% flexibility

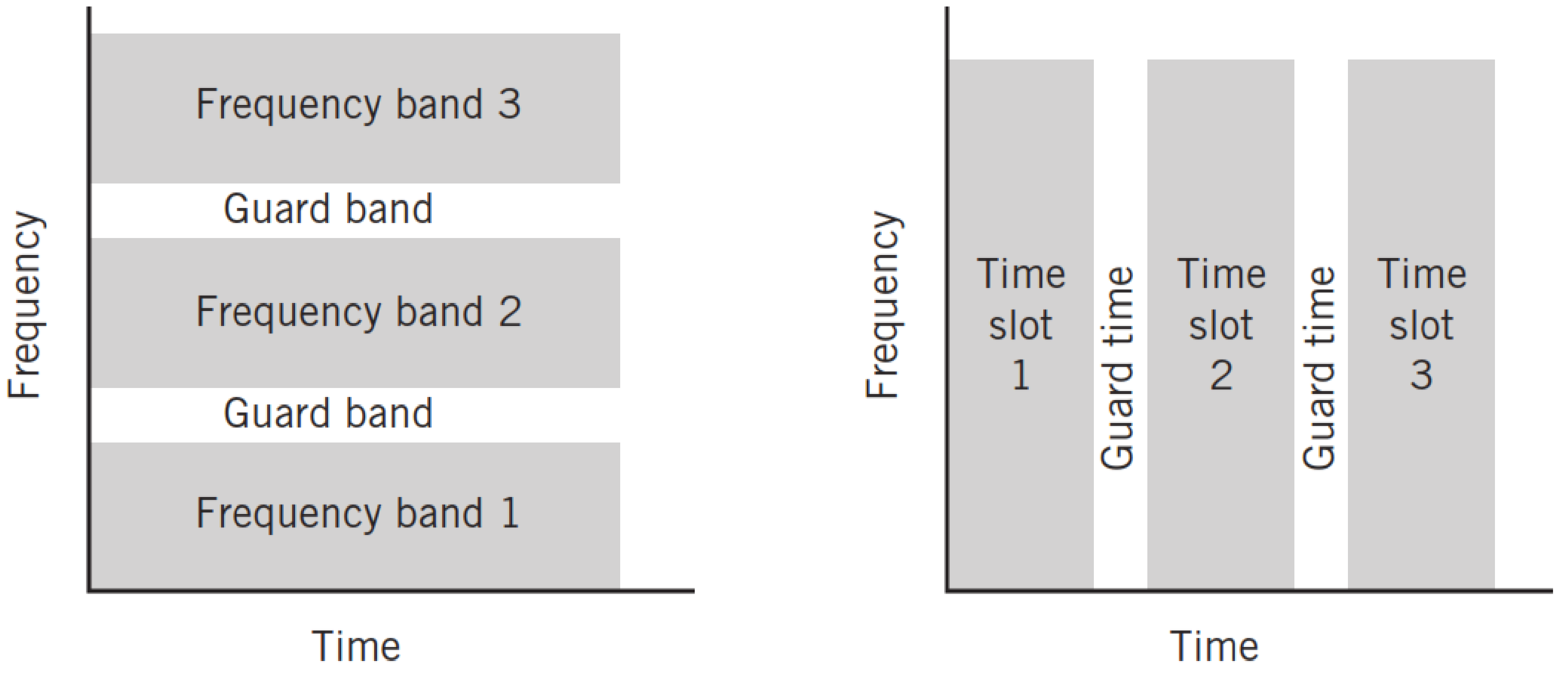
Lecture Outlines

- **Part 1:** Communications concepts
    - Multiple access techniques
    - Computer networks
    - Digital communications
- **Part II:** Review on Fourier analysis of signals and systems
    - The Fourier series
    - The Fourier transform

Multiple access techniques

- Frequency division multiple access (FDMA)
- Time division multiple access (TDMA)
- Code division multiple access (CDMA)
- Space division multiple access (SDMA)

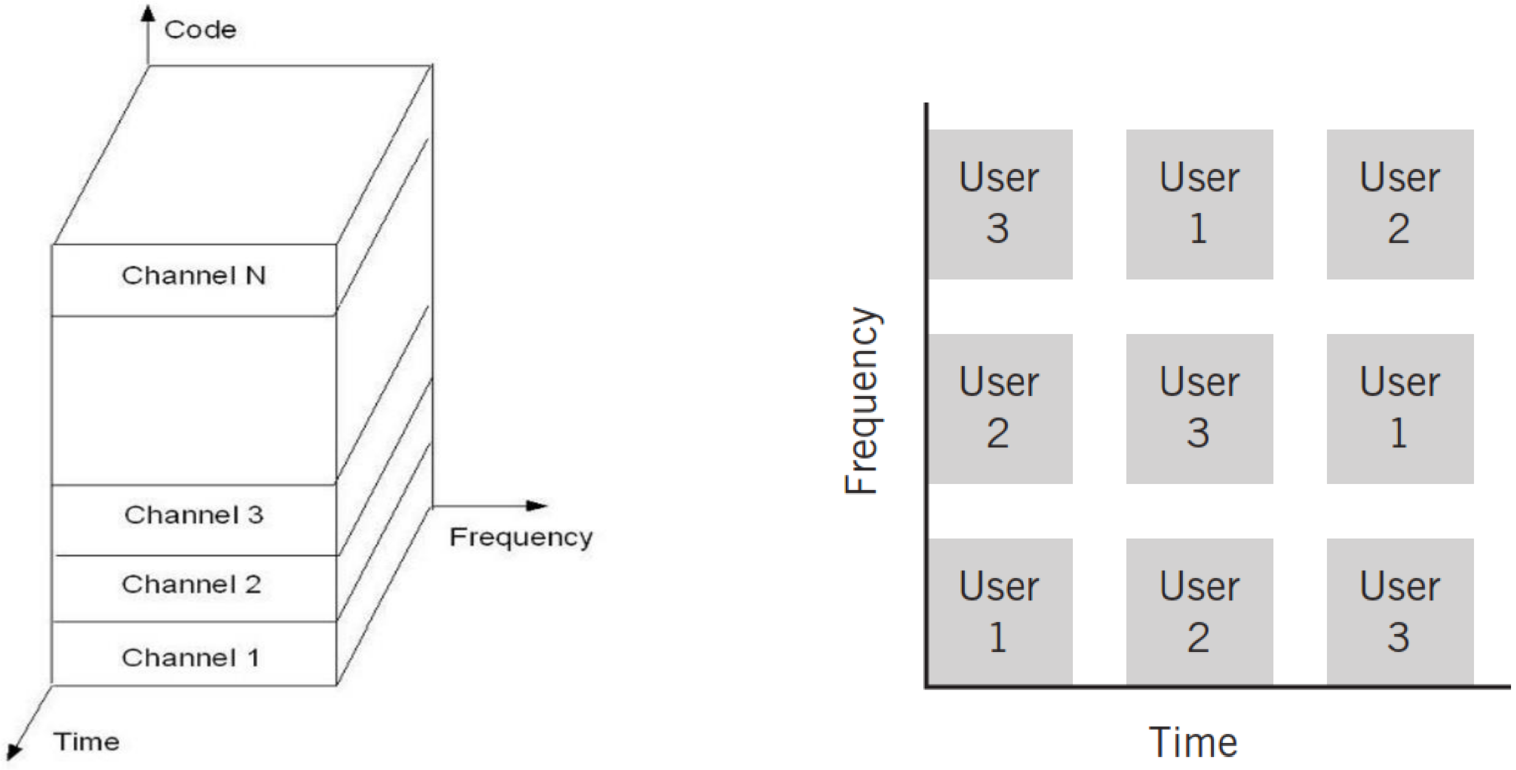
TDMA vs. FDMA



(a) Time division multiple access (TDMA) and (b) frequency division multiple access (FDMA) *

* Image courtesy of *Simon Haykin, "Digital Communication Systems, 1st Edition," Wiley, 2013*

Code division multiple access (CDMA)



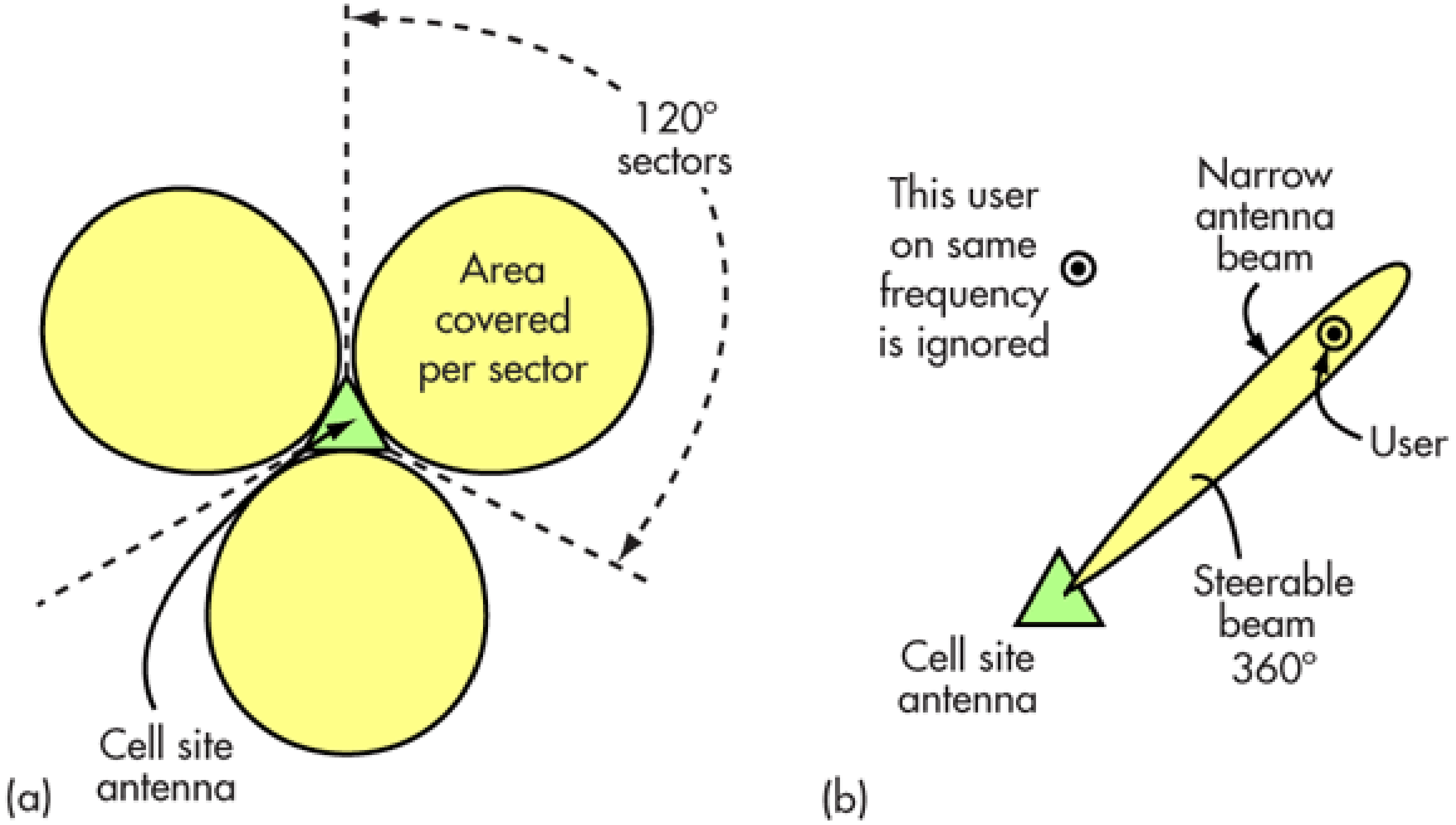
(a)

(b)

(a) Code division multiple access (CDMA) and (b) an example of CDMA: frequency-hop multiple access *

* Image courtesy of *Simon Haykin, "Digital Communication Systems, 1st Edition," Wiley, 2013*

Space division multiple access (SDMA)

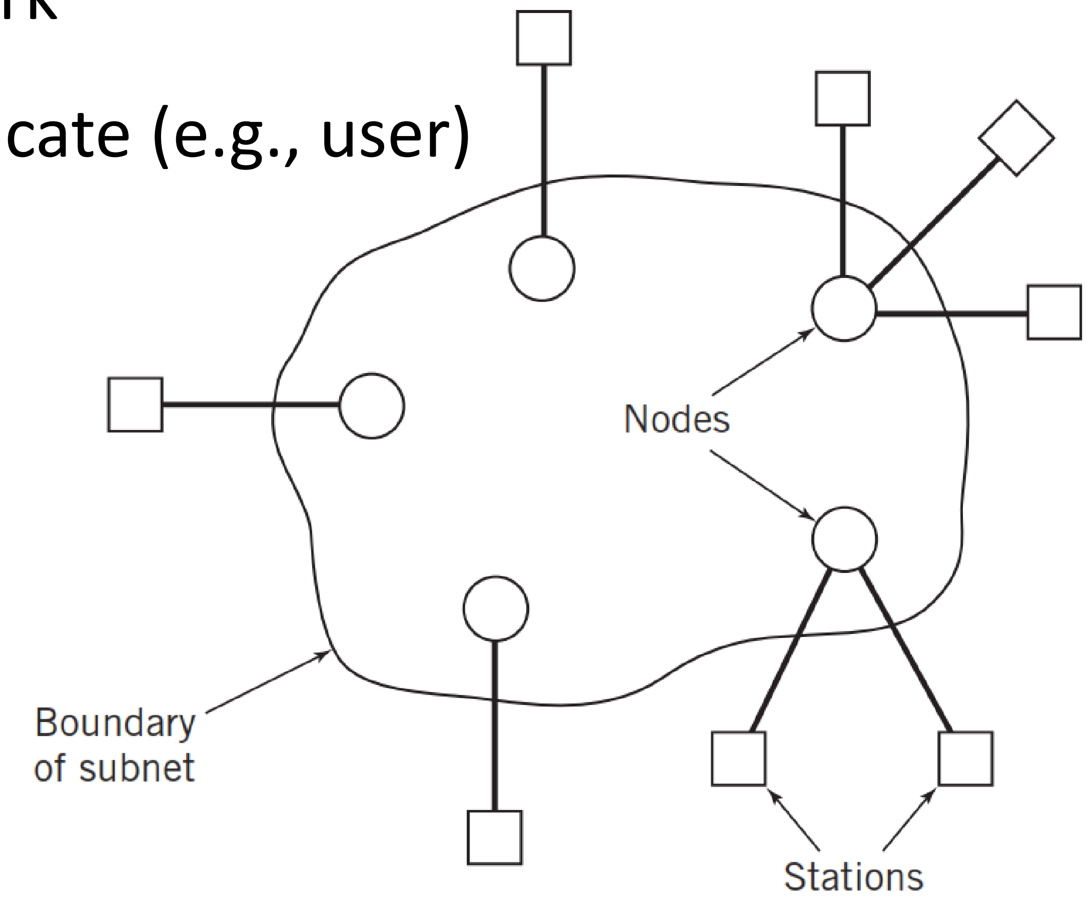


SDMA separates users on shared frequencies by isolating them with directional antennas. Most cell sites have three antenna arrays to separate their coverage into isolated 120° sectors (a). Adaptive arrays use beamforming to pinpoint desired users while ignoring any others on the same frequency (b)*.

* Image courtesy of *L. Frenzel, "Fundamentals of Communications Access Technologies: FDMA, TDMA, CDMA, OFDMA AND SDMA Accommodating multiple users in a single bandwidth is the essence of access methods," 2013*

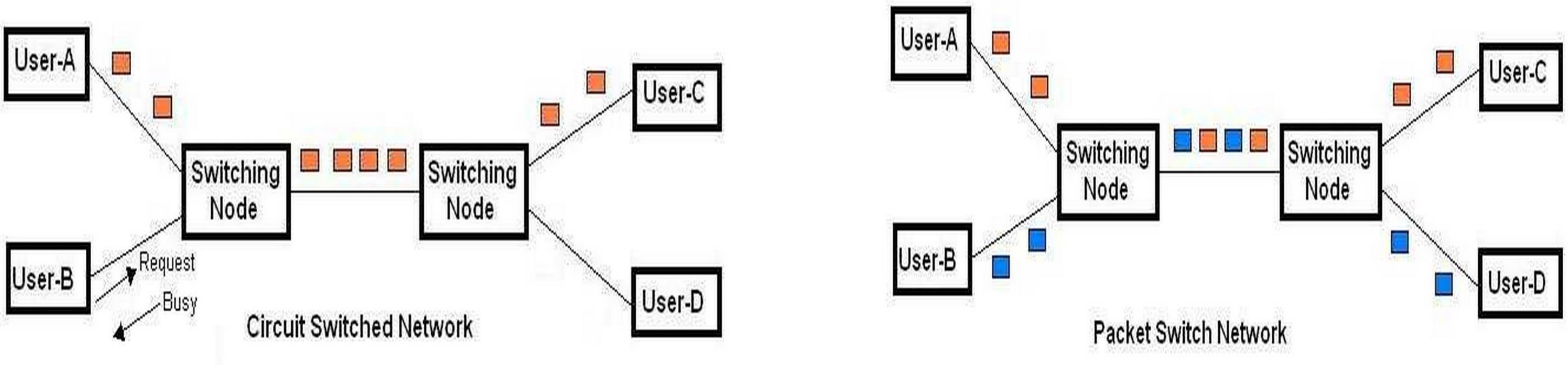
Communication Networks

- Intersection of number of nodes (e.g., processors)
- **Node**: route data through the network
- **Station:** devices wishing to communicate (e.g., user)



* Image courtesy of *Simon Haykin, "Digital Communication Systems, 1st Edition," Wiley, 2013*

Circuit switching vs. packet switching

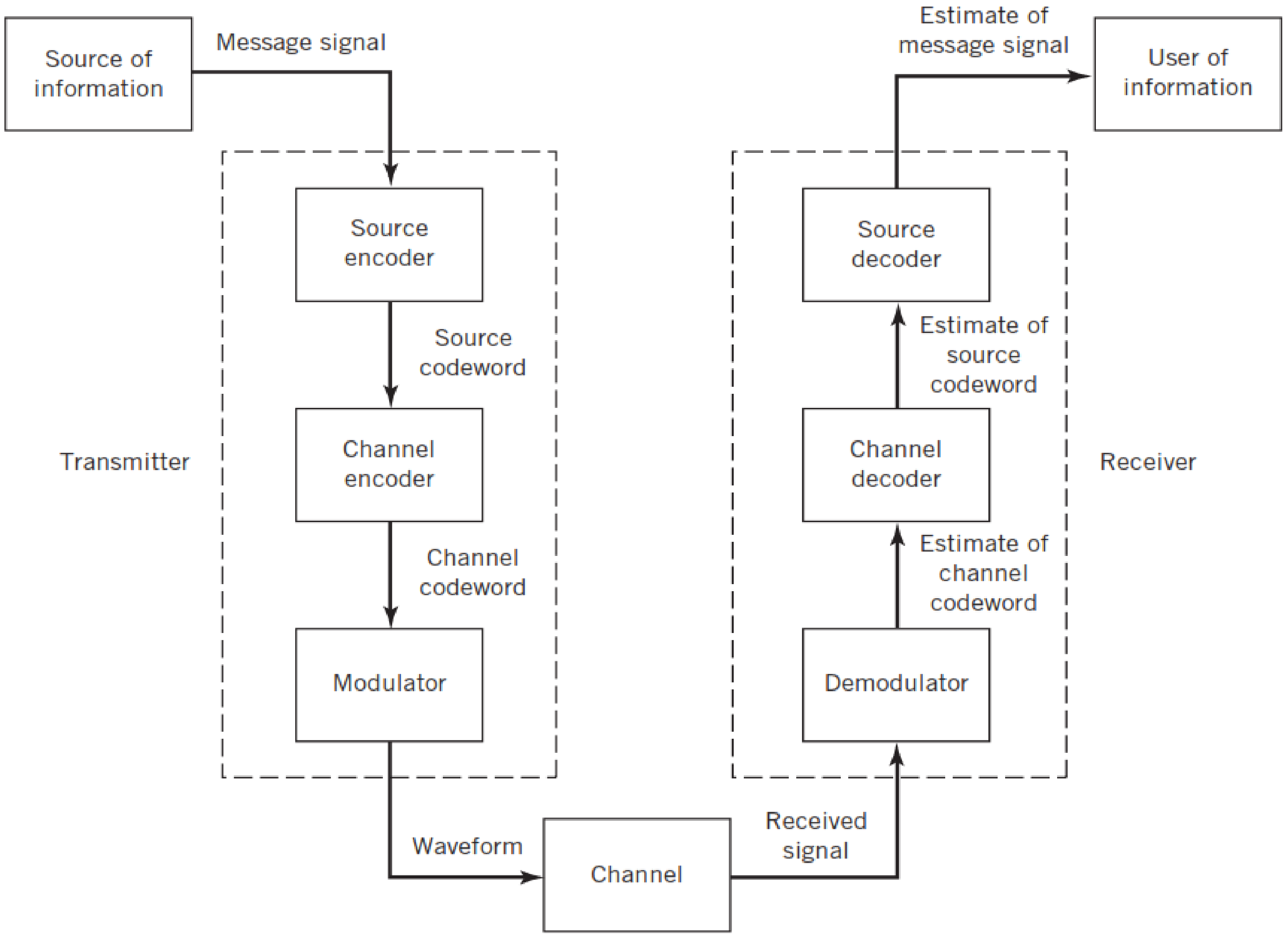


(a) (b)

Circuit switching (link sharing is based on fixed allocation) vs. packet switching (message is divided into segments or packets if necessary, sharing is per demand) *

* Image courtesy of *http://www.rfwireless-world.com/Terminology/circuit-switching-vs-packet-switching.html*

Block diagram of a digital communication system*



* Image courtesy of *Simon Haykin, "Digital Communication Systems, 1st Edition," Wiley, 2013*

Blocks of a digital communication system

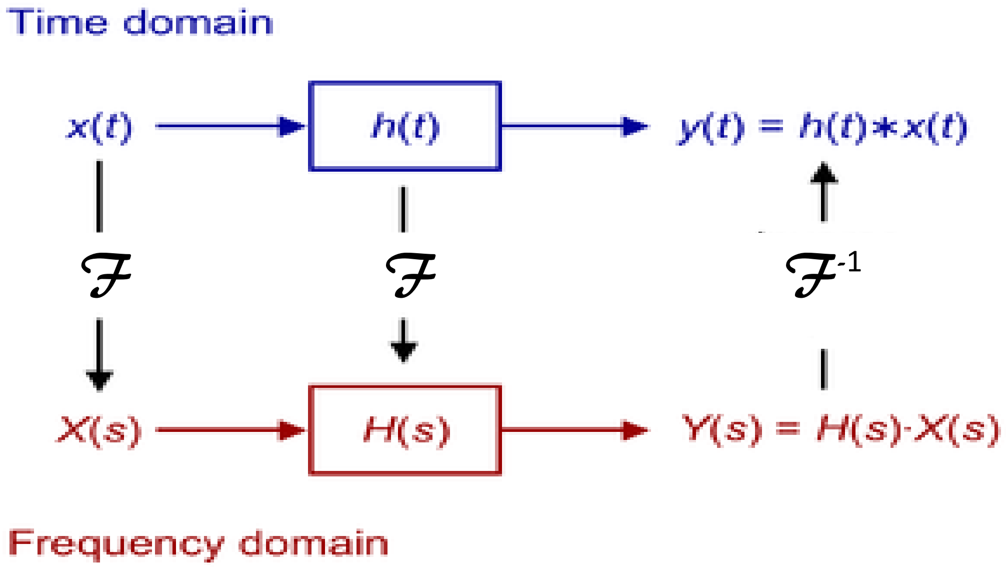
- **Source encoder:** remove redundant information from the message signal, output: source codeword
- **Channel encoder:** produce a new sequence of symbols adding channel controls and error corrections (channel codeword)
- **Modulator:** Represent each symbol of the channel codeword by corresponding analog signal (waveform) among a finite set of analog signals

Lecture Outlines

- **Part 1:** Communications concepts
  - Multiple access techniques
  - Computer networks
  - Digital communications
- **Part II:** Review on Fourier analysis of signals and systems
  - The Fourier series
  - The Fourier transform

Fourier transform

- Provides the mathematical link between the time domain of a signal (waveform) and its frequency domain (spectrum)
- Time and frequency response of a linear time-invariant (LTI) system defined in terms of its impulse response and frequency response, respectively



LTI system

If $x_1(n) \rightarrow \boxed{h(n)} \rightarrow y_1(n)$

and $x_2(n) \rightarrow \boxed{h(n)} \rightarrow y_2(n)$

Then $a_1y_1(n)+a_2y_2(n)=H\{a_1x_1(n)+a_2x_2(n)\}$

Where a1, a2 are scalars

(a)

If $x_1(n) \rightarrow \boxed{h(n)} \rightarrow y_1(n)$

Then $x_1(n-k) \rightarrow \boxed{h(n)} \rightarrow y_1(n-k)$

(b)

A (a) linear and (b) a time invariant system

The Fourier series

- Let $g_{T_0}(t)$ denote a periodic signal, where the subscript T_0 denotes the duration of periodicity.
- By using a Fourier series expansion of this signal, we are able to resolve it into an infinite sum of sine and cosine terms:

$$\boxed{g_{T_0}(t) = a_0 + 2\sum_{n=1}^{\infty}\left[a_n \cos(2\pi n f_0 t) + b_n \sin(2\pi n f_0 t)\right]}$$

- Where $f_0 = \frac{1}{T_0}$ is the fundamental frequency
- a_n and b_n represent the amplitudes of the cosine and sine terms, respectively
- $\cos(2\pi n f_0 t)$ and $\sin(2\pi n f_0 t)$ are called basis functions

Basis functions ($\cos(2\pi n f_0 t)$ and $\sin(2\pi n f_0 t)$)

- Form an orthogonal set over the period T_0
- In that they satisfy three conditions:

$$\int_{-T_0/2}^{T_0/2} \cos(2\pi m f_0 t)\cos(2\pi n f_0 t)\,\mathrm{d}t = \begin{cases} T_0/2, & m = n \\ 0, & m \neq n \end{cases}$$

$$\int_{-T_0/2}^{T_0/2} \cos(2\pi m f_0 t)\sin(2\pi n f_0 t)\,\mathrm{d}t = 0, \qquad \text{for all } m \text{ and}$$

$$\int_{-T_0/2}^{T_0/2} \sin(2\pi m f_0 t)\sin(2\pi n f_0 t)\,\mathrm{d}t = \begin{cases} T_0/2, & m = n \\ 0, & m \neq n \end{cases}$$

Mean value (a_0) and other coefficients

- Integrate over a complete period both sides of the equation:

$$g_{T_0}(t) = a_0 + 2\sum_{n=1}^{\infty}\left[a_n\cos(2\pi n f_0 t) + b_n\sin(2\pi n f_0 t)\right]$$

yields: $a_0 = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_{T_0}(t)\,\mathrm{d}t$

- Other coefficients:

$$a_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_{T_0}(t)\cos(2\pi n f_0 t)\,\mathrm{d}t, \qquad n = 1, 2, \ldots$$

Similarly, we find that

$$b_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_{T_0}(t)\sin(2\pi n f_0 t)\,\mathrm{d}t, \qquad n = 1, 2, \ldots$$

Existence of Fourier series

- A periodic signal can be expanded in a Fourier series if the signal satisfies the *Dirichlet conditions*:

1. The function $g_{T_0}(t)$ is single valued within the interval T_0.
2. The function $g_{T_0}(t)$ has at most a finite number of discontinuities in the interval T_0.
3. The function $g_{T_0}(t)$ has a finite number of maxima and minima in the interval T_0.
4. The function $g_{T_0}(t)$ is absolutely integrable; that is,

$$\int_{-T_0/2}^{T_0/2} \left| g_{T_0}(t) \right| \mathrm{d}t < \infty$$

- From an engineering perspective, however, it suffices to say that the *Dirichlet conditions* are satisfied by the periodic signals encountered in communication systems

Complex exponential Fourier series

$$g_{T_0}(t) = a_0 + 2\sum_{n=1}^{\infty}[a_n\cos(2\pi n f_0 t) + b_n\sin(2\pi n f_0 t)]$$

$$\cos(2\pi n f_0 t) = \frac{1}{2}[\exp(\mathrm{j}2\pi n f_0 t) + \exp(-\mathrm{j}2\pi n f_0 t)]$$

$$\sin(2\pi n f_0 t) = \frac{1}{2\mathrm{j}}[\exp(\mathrm{j}2\pi n f_0 t) - \exp(-\mathrm{j}2\pi n f_0 t)]$$

$$g_{T_0}(t) = a_0 + \sum_{n=1}^{\infty}[(a_n - \mathrm{j}b_n)\exp(\mathrm{j}2\pi n f_0 t) + (a_n + \mathrm{j}b_n)\exp(-\mathrm{j}2\pi n f_0 t)]$$

Let c_n denote a complex coefficient related to a_n and b_n by

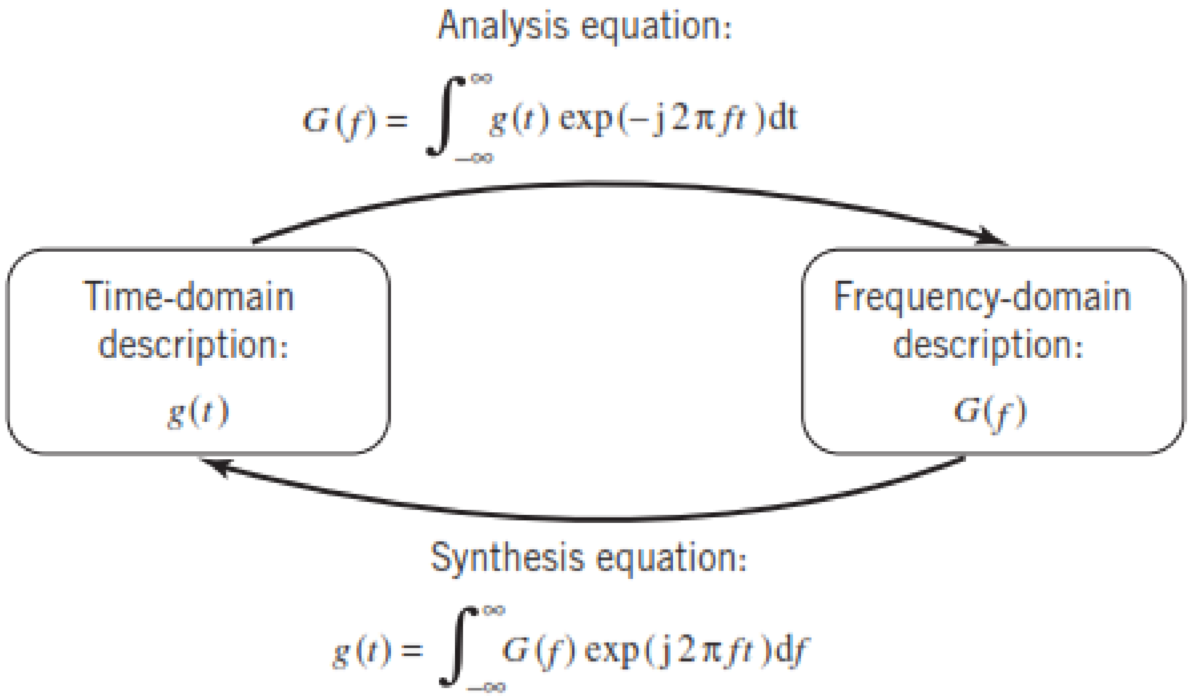
$$c_n = \begin{cases} a_n - \mathrm{j}b_n, & n > 0 \\ a_0, & n = 0 \\ a_n + \mathrm{j}b_n, & n < 0 \end{cases}$$

$$g_{T_0}(t) = \underbrace{\sum_{n=-\infty}^{\infty} c_n\exp(\mathrm{j}2\pi n f_0 t)}_{\text{Inner product of the signal with the basis function } [\exp(-\mathrm{j}2\pi n f_0 t)]}$$

Where $c_n = \dfrac{1}{T_0}\displaystyle\int_{-T_0/2}^{T_0/2} g_{T_0}(t)\exp(-\mathrm{j}2\pi n f_0 t)\,\mathrm{d}t, \qquad n = 0, \pm 1, \pm 2, \ldots$

The Fourier Transform

- Representation of non-periodic signals
- Derivation is based on Fourier series (text book: page 16,17)



Analysis equation:

$$G(f) = \int_{-\infty}^{\infty} g(t)\exp(-j2\pi ft)dt$$

Synthesis equation:

$$g(t) = \int_{-\infty}^{\infty} G(f)\exp(j2\pi ft)df$$

$$g(t) = \lim_{T_0 \to \infty} g_{T_0}(t)$$

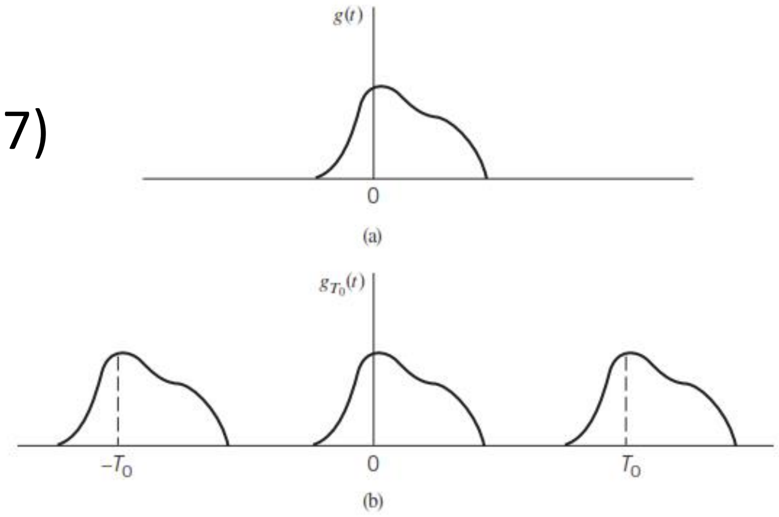
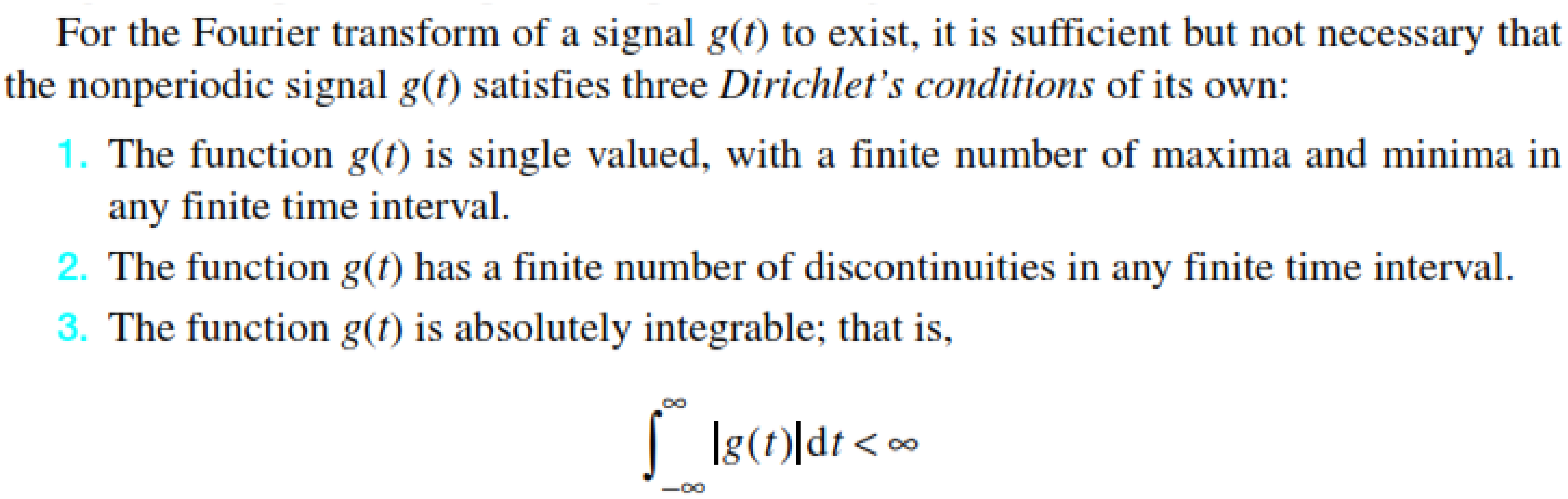


Figure 2.1 Illustrating the use of an arbitrarily defined function of time to construct a periodic waveform. (a) Arbitrarily defined function of time $g(t)$. (b) Periodic waveform $g_{T_0}(t)$ based on $g(t)$.

Existence of Fourier Transform



For the Fourier transform of a signal $g(t)$ to exist, it is sufficient but not necessary that the nonperiodic signal $g(t)$ satisfies three *Dirichlet's conditions* of its own:

1. The function $g(t)$ is single valued, with a finite number of maxima and minima in any finite time interval.
2. The function $g(t)$ has a finite number of discontinuities in any finite time interval.
3. The function $g(t)$ is absolutely integrable; that is,

$$\int_{-\infty}^{\infty} |g(t)| \mathrm{d}t < \infty$$

- Physical realizability is a sufficient condition for the existence of a Fourier transform (e.g., all energy signals are Fourier transformable).

Condition for energy signal: $\int_{-\infty}^{\infty} |g(t)|^2 \mathrm{d}t < \infty$

Fourier Transform

- Provides mathematical tools for measuring the frequency content or spectrum of a signal
- $G(f)$ is the Fourier transform or the spectrum of the signal

$$G(f) = \int_{-\infty}^{\infty} g(t)\exp(-j2\pi ft)\,dt$$

$$g(t) = \int_{-\infty}^{\infty} G(f)\exp(j2\pi ft)\,df$$

- Simple evaluation of magnitude spectrum $|G(f)|$ and phase spectrum $\angle G(f)$

Tables

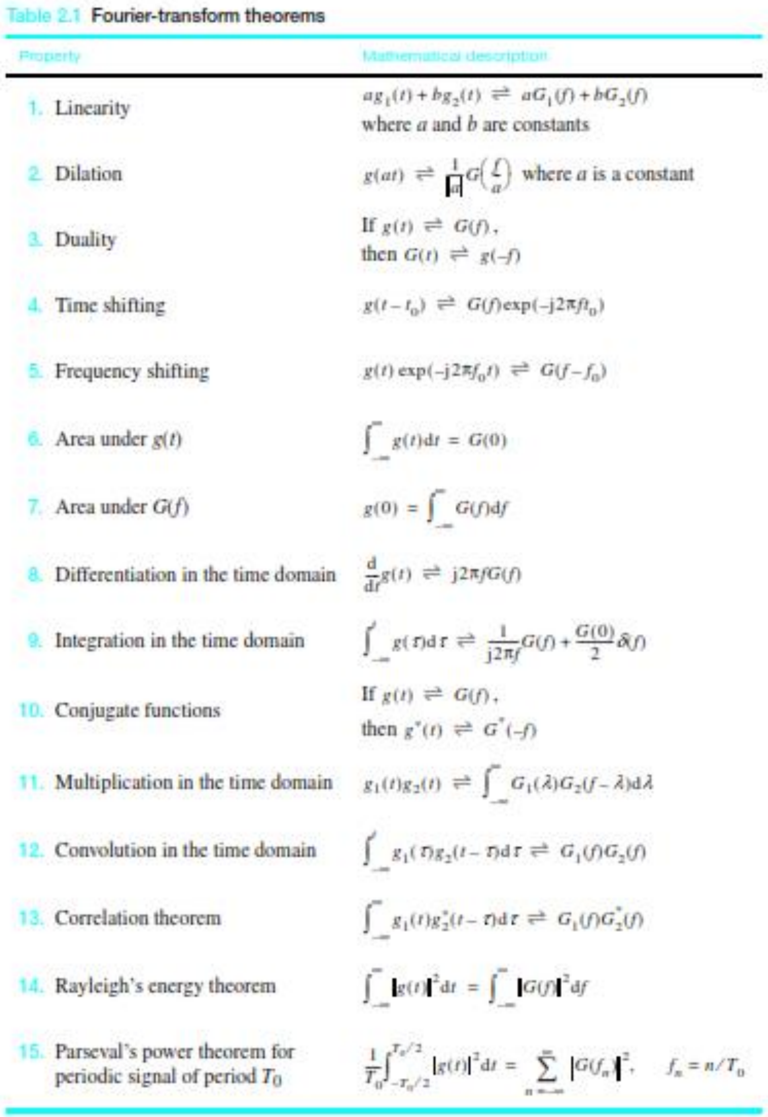


Table 2.1 Fourier-transform theorems

Property	Mathematical description
1. Linearity	$ag_1(t) + bg_2(t) \rightleftharpoons aG_1(f) + bG_2(f)$ where a and b are constants
2. Dilation	$g(at) \rightleftharpoons \frac{1}{\lvert a\rvert}G\left(\frac{f}{a}\right)$ where a is a constant
3. Duality	If $g(t) \rightleftharpoons G(f)$, then $G(t) \rightleftharpoons g(-f)$
4. Time shifting	$g(t-t_0) \rightleftharpoons G(f)\exp(-j2\pi f t_0)$
5. Frequency shifting	$g(t)\exp(-j2\pi f_0 t) \rightleftharpoons G(f-f_0)$
6. Area under $g(t)$	$\int_{-\infty}^{\infty} g(t)\,dt = G(0)$
7. Area under $G(f)$	$g(0) = \int_{-\infty}^{\infty} G(f)\,df$
8. Differentiation in the time domain	$\frac{d}{dt}g(t) \rightleftharpoons j2\pi f G(f)$
9. Integration in the time domain	$\int_{-\infty}^{t} g(\tau)\,d\tau \rightleftharpoons \frac{1}{j2\pi f}G(f) + \frac{G(0)}{2}\delta(f)$
10. Conjugate functions	If $g(t) \rightleftharpoons G(f)$, then $g^*(t) \rightleftharpoons G^*(-f)$
11. Multiplication in the time domain	$g_1(t)g_2(t) \rightleftharpoons \int_{-\infty}^{\infty} G_1(\lambda)G_2(f-\lambda)\,d\lambda$
12. Convolution in the time domain	$\int_{-\infty}^{t} g_1(\tau)g_2(t-\tau)\,d\tau \rightleftharpoons G_1(f)G_2(f)$
13. Correlation theorem	$\int_{-\infty}^{\infty} g_1(t)g_2^*(t-\tau)\,d\tau \rightleftharpoons G_1(f)G_2^*(f)$
14. Rayleigh's energy theorem	$\int_{-\infty}^{\infty} \lvert g(t)\rvert^2\,dt = \int_{-\infty}^{\infty} \lvert G(f)\rvert^2\,df$
15. Parseval's power theorem for periodic signal of period T_0	$\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \lvert g(t)\rvert^2\,dt = \sum_{n=-\infty}^{\infty} \lvert G(f_n)\rvert^2, \quad f_n = n/T_0$

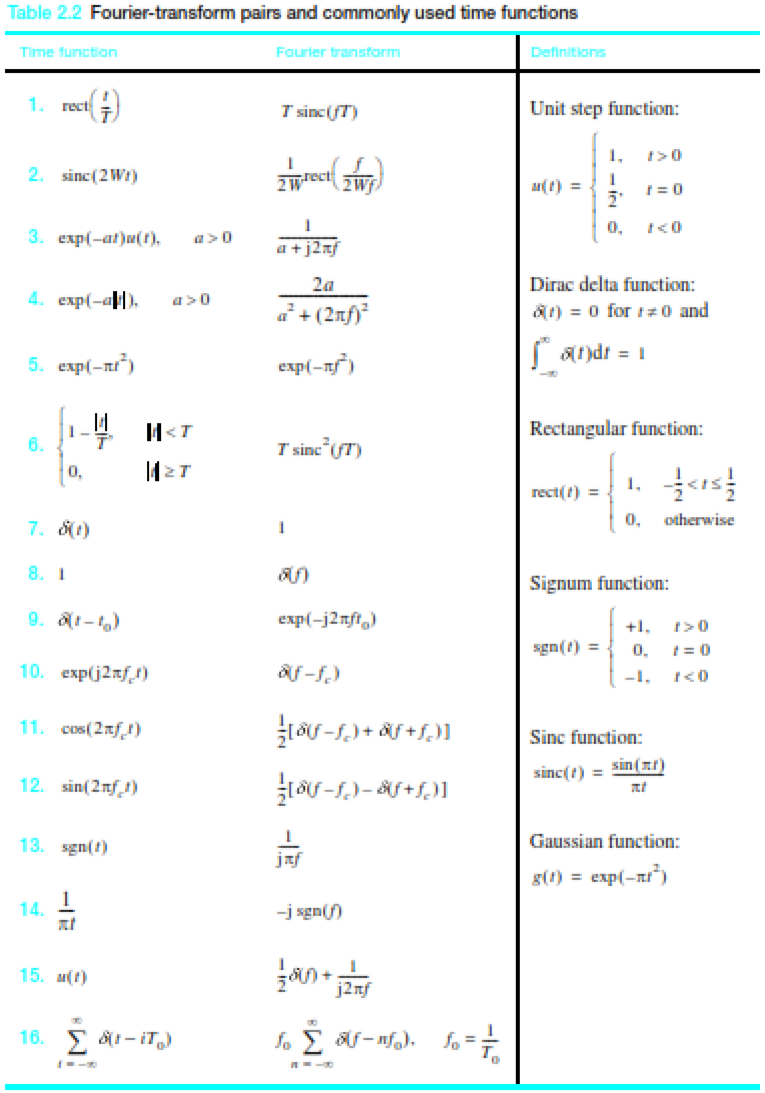


Table 2.2 Fourier-transform pairs and commonly used time functions

Time function	Fourier transform	Definitions
1. $\mathrm{rect}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}(fT)$	Unit step function: $u(t) = \begin{cases} 1, & t > 0 \\ \frac{1}{2}, & t = 0 \\ 0, & t < 0 \end{cases}$
2. $\mathrm{sinc}(2Wt)$	$\frac{1}{2W}\mathrm{rect}\left(\frac{f}{2Wf}\right)$	
3. $\exp(-at)u(t), \quad a > 0$	$\frac{1}{a + j2\pi f}$	
4. $\exp(-a\lvert t\rvert), \quad a > 0$	$\frac{2a}{a^2 + (2\pi f)^2}$	Dirac delta function: $\delta(t) = 0$ for $t \neq 0$ and $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$
5. $\exp(-\pi t^2)$	$\exp(-\pi f^2)$	
6. $\begin{cases} 1 - \frac{\lvert t\rvert}{T}, & \lvert t\rvert < T \\ 0, & \lvert t\rvert \geq T \end{cases}$	$T\,\mathrm{sinc}^2(fT)$	Rectangular function: $\mathrm{rect}(t) = \begin{cases} 1, & -\frac{1}{2} < t \leq \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$
7. $\delta(t)$	1	
8. 1	$\delta(f)$	Signum function: $\mathrm{sgn}(t) = \begin{cases} +1, & t > 0 \\ 0, & t = 0 \\ -1, & t < 0 \end{cases}$
9. $\delta(t - t_0)$	$\exp(-j2\pi f t_0)$	
10. $\exp(j2\pi f_c t)$	$\delta(f - f_c)$	
11. $\cos(2\pi f_c t)$	$\frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$	Sinc function: $\mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$
12. $\sin(2\pi f_c t)$	$\frac{1}{2}[\delta(f - f_c) - \delta(f + f_c)]$	
13. $\mathrm{sgn}(t)$	$\frac{1}{j\pi f}$	Gaussian function: $g(t) = \exp(-\pi t^2)$
14. $\frac{1}{\pi t}$	$-j\,\mathrm{sgn}(f)$	
15. $u(t)$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$	
16. $\sum_{i=-\infty}^{\infty} \delta(t - iT_0)$	$f_0 \sum_{n=-\infty}^{\infty} \delta(f - nf_0), \quad f_0 = \frac{1}{T_0}$	

Questions